Musings on Saint Anselm’s Dilemma
Reflexiones sobre el dilema de San Anselmo

J.-Martín Castro-Manzano
josemartin.castro@upaep.mx

Abstract: In this contribution we suggest two simple contentions: i) that Saint Anselm’s ontological argument, as presented in the Proslogion, only makes sense if embedded in an ordered and bounded ontology; and ii) that an interpretation of the main premise of the argument, within such ontology, produces a dilemma that demands a new revision of the argument. Keywords: Ordered ontology; bounded ontology; greatness.

Resumen: En este trabajo proponemos dos afirmaciones: i) que el argumento ontológico de San Anselmo, como se presenta en el Proslogion, sólo tiene sentido al interior de una ontología ordenada y acotada; y ii) que una interpretación del argumento, dentro de tal ontología, produce un dilema que demanda una nueva revisión del argumento. Palabras clave: Ontología ordenada; ontología acotada; grandeza.

1. Introduction

We share the opinion that the Proslogion should prove as much a philosophical as it does a mathematical work of art, for it presupposes an ontology that requires assumptions close to order theory, namely, assumptions of order and bounds. Following these assumptions, in this short contribution we suggest two simple contentions: i) that Saint Anselm’s ontological argument, as presented in the Proslogion, only makes sense if embedded in an ordered and bounded ontology (§2); and ii) that an interpretation of the

1 Authoritative scholars do not necessarily share this opinion. Anselm Stolz (1967), for instance, suggested the Proslogion is a piece of mystical theology. Karl Barth (1960) offered a more conservative assessment in that St. Anselm’s work is fundamentally theological. Étiene Gilson (1934), however, argued the Proslogion is not philosophy any more than it is theology or mysticism, and thus suggested the label gnosticisme chrétien. We believe, nevertheless, that the Proslogion has some resemblance with contemporary classical mathematics for its central argument behaves like a theorem of uniqueness and existence as they usually appear in classic mathematics (cf. Oppenheimer and Zalta, 1991).
argument, within such ontology, produces a dilemma that demands a new revision of the argument (§3).

2. Remarks on the Anselmian ontology

For the purposes of this contribution we define an ontology by a pair \( \langle D, R \rangle \) where \( D \) is a non-empty domain and \( R \) is a relation defined on \( D \). In order to describe the Anselmian ontology, \( O_A \) for short, we consider Pros. II, and Pros. III—which contain subtle versions of the ontological argument (cf. Malcolm, 1961) —, along with Pros. V, Pros. XV, and Mon. I, and Mon. III.

From these passages we can infer, given the uses of \textit{maius}/\textit{melius} (cf. Brecher, 1974), that the Anselmian ontology is defined by the pair \( O_A = \langle B, \geq \rangle \) where \( B \) is a non-empty bounded set of beings and the \textit{maius}/\textit{melius} relation, namely \( \geq \), is a partial order relation on \( B \). We can summarize these features with the following facts:

\textbf{Fact 1.} \( O_A \) is a partially ordered ontology.

\textbf{Fact 2.} \( O_A \) is a bounded ontology.

These facts provide evidence to the claim that St. Anselm’s ontological argument only makes sense if embedded in an ordered and bounded ontology\(^2\). Indeed, we could not account for the \textit{maius}/\textit{melius} relation without an order relation (Fact 1); and we could not account for the notion of “that than which nothing greater can be conceived” (or “the greatest being” for short) without (upper) bounds (Fact 2). However, the very notion of “the greatest being” is not naive and requires further attention because, for it to make sense, \( O_A \) must comply with two further remarks:

\textbf{Remark 1.} \( O_A \) has at least one maximal being.

\textbf{Remark 2.} \( O_A \) has at most one greatest being.

\(^2\) These facts about the Anselmian ontology can also be inferred from a “hierarchy of being” that can be found in the Neoplatonic assumption that, despite all beings share existence, higher beings have a greater proportion of being. This idea allows us to understand what thinkers like St. Augustine, St. Anselm himself, Boethius, and Dante were doing (McMahon, 2006). McMahon is of course correct in that, although the first three did not have the \textit{analogia entis} as an explicit intellectual instrument, the hierarchy of being was a cultural commonplace, especially for St. Anselm, who was a thinker with a Platonic/Augustinian imprint.
Consider, for Remark 1, that a maximal being in a bounded and partially ordered ontology, such as the Anselmian, would be a being $b_x$ in the domain for which there is no being $b_y$ such that $b_y \geq b_x$. So, take an arbitrary being $b_i$ in $O_A$. Suppose, for reductio, that $b_i$ is not maximal. Then we can find another being, say $b_{i+1}$ (i.e. a next being), s.t. $b_{i+1} > b_i$ (i.e. a next being that is greater than the previous one). Now, if $b_{i+1}$ is not maximal itself, we will find yet another being $b_{i+2}$ s.t. $b_{i+2} > b_{i+1} > b_i$. If we continue looking for greater beings in this way, we will have to stop eventually at a bound, call it $g$, corresponding to a being $b_g$ since $O_A$ is bounded above, by Fact 2. Therefore, it is impossible to have $b_i > b_g$ for any $i$ in $O_A$. Hence, there is at least one being $b_g$ that is maximal in $O_A$.

Additionally, for Remark 2, since $O_A$ is a partially ordered ontology, by Fact 1, we also have at most one greatest being. A greatest being in a bounded and partially ordered ontology would be a being $b_g$ in the domain such that for all $b_x, b_y \geq b_x$. So Remark 2 is clear, for suppose there are two greatest beings $b_g$ and $b_k$ in $O_A$, then $b_k \geq b_g$ and $b_g \geq b_k$. But then, by the antisymmetry of the order relation, $b_g = b_k$, that is to say, there is at most one greatest being in $O_A$, namely, $b_g$.

Together, these Facts and Remarks make sense of the following proposition:

**Proposition 1.** If $b_g$ is both the maximal and greatest being in $O_A$, then $b_g$ is the supremum in $O_A$.

Proposition 1 clarifies the meaning of the expression “the greatest being” and specifies that the domain of beings $B$ should behave like the closed-to-the-right interval $[b_0, b_g)$, for any beings $i$ in $B$, where $b_g$ denotes the supremum; otherwise $O_A$ would comply with Fact 1 (order) but not with Fact 2 (boundedness), which would imply the failure of Remarks 1 and 2.

At this point, some numerical examples will help clarify this last proposition. Consider the open interval $(-10, 10)$. This interval is bounded above, for instance, by the numbers 11, 12, ..., 100. This is because upper bounds need not belong to the given set. But in the interval $(-10, 10]$, for example, 10 is the least upper bound and, since it belongs to the set, it is also the supremum. Thus, the interval $(-10, 10)$ has a least upper bound (number 10).

---

3 This interval could also be represented, without loss of generality, by the interval $(b_0, b_g]$, for the important bound is the one on the right.
but no supremum; while \((-10,10]\) has both a least upper bound and a supremum, namely, number 10. The interval \((-10,+\infty)\), on the other hand, is not bounded above and it has no supremum.

To further illustrate Proposition 1 let us consider a more concrete example. Imagine we have a domain of books, say a library, of different height, say \(b_1\), \(b_2\) and \(b_3\) ordered with respect to their height in such a way that \(b_3 \geq b_2 \geq b_1\). Clearly, we can identify “the highest book” in the library, that is to say, that book “than which no higher book can be conceived” in that library, namely, \(b_3\). Now, what must be stressed in this situation is that the only way in which it makes sense to talk about “the highest book” is if the domain is both ordered and bounded (i.e. if Facts 1 and 2 obtain). But notice that if the domain of books is bounded, then \(b_3\) is bounded itself, since the bound of the domain is determined by \(b_3\)’s actual height, for otherwise the domain would be unbounded. So, for instance, the intervals \((-10,10]\) and \((-10,10)\) are clearly bounded above because 10 is bounded itself; had the number 10 no bounds of its own, both intervals would behave like the unbounded interval \((-10,+\infty)\), which would require the number 10 to behave like +\(\infty\), which is absurd, for +\(\infty\) is not even a number. Likewise, in the case of the books, had the book \(b_3\) no bounds of its own, the domain of books would look like the unbounded \((-10,+\infty)\), which would require the book \(b_3\) to behave like +\(\infty\), which is also absurd, for +\(\infty\) denotes the interval’s behavior, not an actual book or entity. Hence, the book \(b_3\) is a bound of the domain because it has a bound of its own, say its actual height denoted by the definite number 3. This is an important step for what comes next because, suppose, for reductio, that \(b_g\) is the supremum of \(O_A\) but is not bounded itself. If \(b_g\) is the supremum of \(O_{A'}\), the domain \(B\) must behave like a closed-to-the-right interval, say \((b_g, b_g]\). But if \(b_g\) is not bounded itself, \(b_g\) would behave like +\(\infty\), in which case \(B\) would behave like the open interval \((b_g, +\infty)\), but that would contradict Fact 2. We call this the bounded-bound condition. The bounded-bound condition can be best “viewed” with the aid of the following diagrams:
From left to right, the first diagram shows a representation of a bounded space as a ball of finite radius so that for all $b_x, b_g \geq b_x$, i.e., the distance $d(b_x, b_y) \leq b_g$ for any beings $b_x$ and $b_y$. This represents a situation that considers that we may admit a possible infinite set of beings in $B$, but $B$ is still bounded by $b_g$. The second diagram represents $O_A$ as a bounded chain. In both diagrams the domain is bounded because it is true that for all $b_x, b_g \geq b_x$.

3. A loop and a dilemma

At this point, nothing seems to be too controversial, especially regarding the previous examples because, for one, the notion of “the highest book” is scarcely problematic. However, what happens if we think, not of “the highest book”, but of “the greatest being”? Let us entertain this question for a moment. Suppose $b_g$ is the greatest being, according to $O_A$. Then, $b_g$ is precisely the supremum, by Proposition 1. Now, if $b_g$ is the supremum, $b_g$ is bounded itself, by the bounded-bound condition. However, if $b_g$ is bounded in itself, it has some sort of limit, namely the limit defined by its own bound (as in the example of the book, it’s very height was its own bound); but if $b_g$, “that than which nothing greater can be conceived”, has bounds of any sort, could not we think of something greater than $b_g$, namely, something not constrained by the bound defined by the limit $g$? After all, why would the greatest being have bounds of any sort?

In other words, if we say $b_g$ is the supremum, then we would have to admit that $b_g$ has some sort of own bound or limit. If this is the case, however, we could think of a being beyond that bound, in which case we could extend $B$ indefinitely (just as we can think of a number greater than 10 when considering the interval $(−10,10]$). On the other hand, if we say there is some being greater than any other being, we would have to admit that there is no supremum whatsoever. If this is the case, nevertheless, we could extend $B$ indefinitely and then $B$ would be unbounded; but this would be odd, because then there would be no supremum at all; but if there is a supremum, then such being would have to be bounded, but then we could think of a being beyond that bound again, namely, a greater being, and then we could extend $B$ again, and so on and so forth. However, this turns out to be kind of paradoxical because it leads to evident inconsistencies regarding the notion of “the greatest being”. To visually explain this situation, consider the next flow chart:
Musings on Saint Anselm’s Dilemma

The flow chart above starts by asking whether greatness has bounds of any sort. If greatness has no bounds, it does not make sense to talk about “the greatest being”. But, on the other hand, if greatness has bounds, the notion of “the greatest being” makes sense, but then we can ask if we can think of something above those bounds; and in case we can, then it does not make sense to talk about “the greatest being” again; and in case we cannot, we get into a loop and we return to the original question that starts the flow chart.

With this loop schema in mind, we can now expound what we call “Saint Anselm’s dilemma”:

1. To avoid the loop, either we maintain the assumption that “$b$ is the greatest” while dropping the notions of order and bounds, namely $O_A$; or we maintain such notions, namely $O_A$, without dropping the assumption that “$b$ is the greatest”.

2. The problem with the first horn of the dilemma, of course, is that we cannot abandon $O_A$ without dropping the thesis that “$b$ is the greatest”, since such abandonment would render the very proposition “$b$ is the greatest” meaningless (due to Proposition 1), and then the Anselmian ontological argument would be unsound (because of an ill-defined premise).
3. But then we should consider the other horn of the dilemma and drop the claim that "\( b_g \) is the greatest" while keeping \( O_{A'} \), but this strikes us as odd because the thesis that "\( b_g \) is the greatest" seems to be both the main premise of the Anselmian ontological argument and a natural consequence of \( O_A \) (due to Remarks 1, 2, and Facts 1, 2), but then the Anselmian argument would be unsound (because it would be self-defeating).

Since, in both cases, the Anselmian ontological argument turns out to be unsound, but we know the argument is logically valid (cf. Oppenheimer and Zalta, 1991), the problem must come from the layman use of the notion of "the greatest being", and so this notion has to be recast with more attention in order to avoid any inconsistencies or loops. For instance, an obvious objection that undercuts the previous remarks is the use of the "bounded-bound condition", since it seems to be an artificial and controversial add-on to Saint Anselm’s original proposal. Granted. This so-called bounded-bound condition is not to be found within the Anselmian corpus. However, since the notion of "the greatest being" produces these artificial, yet troublesome considerations, the bounded-bound condition must not be discarded altogether as an alien one. It may certainly be artificial, but it is also not strange to \( O_{A'} \) due to Remarks 1, 2, and Facts 1, 2.

4. Concluding remarks

In this contribution we have suggested two simple contentions. The first one, rather typical, states that Saint Anselm’s ontological argument, as presented in the Proslogion, only makes sense if embedded in an ordered and bounded ontology; the second one, which we think is rather controversial, focuses on an interpretation of the argument within such ontology that produces a dilemma that demands a new revision of the argument due to the ambiguity of the notion of "the greatest being".

The dilemma shows, we think, that unrevised assumptions of order and bounds can render some propositions unsound if we do not handle them with care. And this is an important and delicate issue, especially if we consider, for instance, any conception of God as a being defined in terms of greatness⁴, for such greatness would have to be embedded in an ordered

---

⁴ Whether it is done for worship-justification reasons (cf. Niemeyer Findlay, 1948) or proof-justification purposes (cf. Plantinga, 1974).
and bounded ontology in order to make any sense; but then we would need to revise our order assumptions more carefully, because the notion of greatness, if not managed carefully and precisely, might lead to strange loops where none should be.

References


Recepción: 05.01.2019
Aceptación: 25.02.2019